A', A'', B, C, C', C'' = constants in the trial extra stress profile
C_d = drag coefficient
d_{ij} = component of the rate-of-deformation tensor
D, E, E', E'' = constants in the trial extra stress profile
E = work function
E_c = complementary work function
$\mathcal{E}$ = energy dissipation rate
f = Fanning friction factor, $\Delta Pa/PV_o^2L$
$\mathbf{f}$ = body force
F, F', F'' = constants in the trial extra stress profile
F_d = drag force on the sphere
I = unit tensor
II = second invariant of the rate-of-deformation tensor
II_τ = second invariant of the extra stress tensor
J_1, J_2 = integrals defined by (24) and (38)
K = consistency index
n = flow behavior index
$\mathbf{n}$ = normal vector in surface integrals
p = pressure
$\mathbf{r}$ = dimensionless radius vector
$\mathbf{R}$ = radius vector
r_∞ = dimensionless radius of the free surface
R_∞ = radius of the free surface
Re_p = Reynolds number, $V_o^{2-n}(2a)^n\,\rho/K$
S = bounding surface of the flow domain
S_t = the part of the bounding surface on which the stress is explicitly stated
S_v = the part of the bounding surface on which the velocity is explicitly stated
$tr(\mathbf{A})$ = trace of matrix $\mathbf{A}$
$\mathbf{T}$ = stress tensor
$\mathbf{v}$ = velocity vector
V = volume domain of the flow
V_o = superficial velocity of the fluid in the assemblage
x = reciprocal r
x_∞ = reciprocal r_∞
Y_P = $C_dRe_p/24$
Y_{UB}, Y_{LB} = upper and lower bounds on Y_P
z = $\cos\theta$

Greek Letters

δ_{ij} = Kronecker delta
ϵ = bed voidage
η = fluid viscosity
ρ = fluid density
σ = constant in the stream function profile
$\boldsymbol{\tau}$ = extra stress tensor
ϕ = azimuthal coordinate
Φ = body force potential
ψ = stream function

Symbols

∇ = operator del
$\cdot$ = dot product

Subscripts

r, θ = radial and angular components

Superscripts

— = dimensionless quantity
* = quantity derived from trial profile

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A Comprehensive Correlating Equation for Forced Convection from Flat Plates

A correlating equation was developed which provides a continuous representation for all *Pr* and *Re*. Different constants are suggested for the local and mean Nusselt numbers and for uniform wall temperature and heating. These constants are based on the best available theoretical and experimental results.

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SCOPE

Correlating equations are more convenient and accurate for design calculations, particularly with computers, than graphs or tabulations. Prior correlating equations have on the main been limited to one regime of motion and to a narrow range of Prandtl number. The effects of boundary conditions and upstream turbulence have generally been ignored. The experimental data are surprisingly limited in range and demonstrate considerable scatter, probably owing to undefined secondary variables. The points of transition from laminar to transitional motion and from transitional motion to turbulent motion have not been correlated satisfactorily.

The objective of this study has been to develop a single correlating equation for all regimes of flow (all Re), all Pr, all boundary conditions, and all points of transition from laminar to turbulent flow. The available experimental and theoretical values for the conductive, laminar boundary layer, transitional and turbulent regimes were first examined critically and correlated individually. The Churchill-Usagi equation, which provides a flexible method of interpolation between limiting solutions, was then used to tie these individual correlations into a single overall correlation.

The resulting expression is quite suitable for incorporation in algorithms for design calculations. The arbitrary constants in the correlating equation can be readily modified as better theoretical and experimental values become available.

CONCLUSIONS AND SIGNIFICANCE

The correlating equation presented herein provides a continuous representation for Nu for a complete range of Re and Pr. Different values for some of the constants in the equation are suggested for the local and mean Nusselt numbers and for uniform wall temperature and uniform heating. The values of some of these constants are uncertain owing to limited or scattered data, and it may be appropriate to revise them as additional or better data become available. Even so, the single overall correlation is more successful than most of the prior correlations for a limited range because of the varying dependence provided on Re and Pr. The chosen exponents in the Churchill-Usagi expression are quite arbitrary, but the correlation is quite insensitive to these values. The equation incorporates one arbitrary constant corresponding to the point of transition from laminar motion. The value of this constant appears to depend on extraneous variables such as upstream turbulence, surface roughness, and the history of the flow, as well as on the Prandtl number.

The correlating equation can be interpreted for component transfer by simply substituting Sh for Nu and Sc for Pr.

Correlating equations such as this are more convenient and accurate for calculations with computers than the traditional graphs, tabulations, and piecemeal equations.

Forced convection to a flat plate is of intrinsic and practical interest both directly and as a limiting case for flow through and along tubes. Even so, the prior work is somewhat limited. The computed values of Nu_x for the laminar boundary layer regime have been reviewed and correlated by Ozoe and Churchill (1973*a*, 1973*b*) for both uniform wall temperature and uniform heating. Dennis and Smith (1966) computed values of $\overline{Nu}$ for Re below the range of applicability of boundary layer theory. Extensive experimental results for Nu_x and $\overline{Nu}$ have been presented by Žukaukas and Šlančiauskas (1973) for a variety of fluids. More limited results have been obtained by a number of other investigators.

Whitaker (1972) proposed the following correlating equation for $Re > 2 \times 10^5$ and $Pr > 0.7$:

$$\overline{Nu} = 0.036\,Pr^{0.43}\,(Re^{0.8} - 9\,200)\,(\mu_\infty/\mu_s)^{1/4} \qquad (1)$$

This expression postulates a sudden transition from laminar to turbulent motion at $Re = 2 \times 10^5$. Other proposed correlating equations appear to be limited to the laminar, transition, or turbulent regime only.

The objective of this work has been to develop correlating equations for Nu_x and $\overline{Nu}$ in terms of Pr and Re by using the model of Churchill and Usagi (1972).

LAMINAR REGIME

Churchill and Ozoe (1973*b*) correlated the computed values for the local Nusselt number in the laminar boundary layer regime on a plate at uniform temperature with the expression

$$Nu_x = 0.3387\phi^{1/2} \qquad (2)$$

where

$$\phi = RePr^{2/3}/[1 + (0.0468/Pr)^{2/3}]^{1/2} \qquad (3)$$

Churchill and Ozoe (1973*a*) correlated the computed values for uniform heating with an equation of the same form but with the coefficients 0.4637 and 0.02052 in place of 0.3387 and 0.0468.

Integrating the heat transfer coefficient for uniform wall temperature from 0 to x, we get the coefficient 2(0.3387) = 0.6774 for $\overline{Nu}$. The definition of $\overline{Nu}$ for uniform heating is arbitrary. The choice of ΔT at $x/2$ yields a coefficient for $\overline{Nu}$ equal to $2^{1/2}(0.4637) = 0.6558$, thus differing only 3% from that for uniform wall temperature. The difference in the central value (divisor) of Pr is significant only for $Pr << 1$. Hence, the values for uniform wall temperature can be used for uniform heating without serious error in the case of $\overline{Nu}$.

The values of $\overline{Nu}$ computed by Dennis and Smith (1966) for small Re and uniform surface temperature are higher than those from boundary layer theory. Based on the experience of Churchill and Chu (1975) for free convection to a vertical plate, the test expression

$$\overline{Nu}^n = \overline{Nu}_o^{\,n} + (0.6774\phi^{1/2})^n \qquad (4)$$

was postulated to represent these values. Values of $n = 1$ and $Nu_o = 0.45$ were chosen by trial and error. The success of this representation is indicated in Figure 2. The small discrepancies are due to the failure of the computed values of Dennis and Smith to converge exactly to boundary layer theory as $Re \to \infty$. It follows that

$$Nu_x = Nu_o + 0.3387\phi^{1/2} \qquad (5)$$

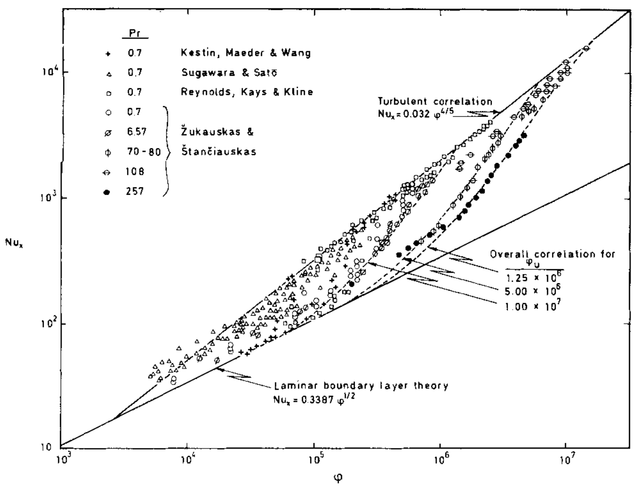

Fig. 1. Correlating equations for local Nusselt number in forced convection from flat plates.

TURBULENT REGIME

The various experiments all indicate that Nu_x and $\overline{Nu}$ are approximately proportional to $Re^{0.8}$ in the turbulent regime. The dependence on Pr is not defined clearly by theory or experiment. The older correlations generally postulate a proportionality to $Pr^{1/3}$. Zukauskas and Slančiauskus (1973) fitted their data for $0.7 < Pr < 257$ with an exponent of 0.43. Hanna and Myers (1962) recommended a coefficient of 0.5 for $Pr > 1$ based on a semi-theoretical analysis and a review of prior data and analyses.

If Nu_x is postulated to be a function of $RePr$ as $Pr \to 0$, for turbulent as well as laminar flow the expression

$$Nu_x = B\,[Re^{1/2}Pr^{1/3}/[1 + (C/Pr)^{2/3}]^{1/4}]^m \tag{6}$$

is suggested. In the absence of data for small Pr, the coefficient C is postulated to be the same as for laminar flow. This concept was used successfully by Churchill and Chu (1975) for free convection. If the experimental exponent of 0.8 is used for Re, then $m = 2(4/5) = 8/5$, and the exponent of Pr in the numerator approaches 8/15 as $Pr \to \infty$. The local rate data of Žukauskus and Šlančiauskas suggest a coefficient $B = 0.032$. The final expression is then simply

$$Nu_x = 0.032\phi^{4/5} \tag{7}$$

TRANSITION REGIME

The point of transition from laminar to turbulent flow is dependent on many extraneous variables such as free stream turbulence, surface roughness and history, as well as on Pr and Re. The dependence of Nu_x on Re in the transition region is roughly proportional to the 3/2 power for all conditions as suggested by the various sets of data in Figure 1. If the same relationship between Re and Pr as for laminar and turbulent flow is postulated for the transition regime, the expression

$$Nu_x = E\,\phi^{3/2} \tag{8}$$

is suggested. As indicated in Figure 1, this model does not completely correlate the dependence on Pr, perhaps because of the presence of extraneous variables. Hence a value of E must be chosen for each set of self-consistent data for a particular fluid and set of conditions. It is perhaps more convenient to introduce the lower critical value of the function $\phi = \phi_l$ for transition from laminar flow, as defined by the intersection of Equations (2) and (8). It follows that

$$E = 0.3387/\phi_l \tag{9}$$

Hence

$$Nu_x = 0.3387\phi^{1/2}\,(\phi/\phi_l) = 0.3387\phi^{1/2}\,(Re/Re_l) \tag{10}$$

MULTIPLE REGIMES

Equations (5), (7), and (10) can be used individually for the laminar, transition, and turbulent regimes. However, the shift from one regime to another is actually smooth instead of discontinuous in the derivative. An overall equation in the form proposed by Churchill and Usagi (1972) can therefore be used to construct an even better representation.

The overall representation was developed by first combining the equations for the transition and turbulent regimes and then combining this expression with the equation for the laminar regime. (The alternative order of combining fails in the limit of no transition regime.) The test expression for the transition and turbulent regimes can be written in terms of ϕ_l as

$$Nu_x^p = (0.3387\phi^{3/2}/\phi_l)^p + (0.032\phi^{4/5})^p \tag{11}$$

Any large, negative value of p provides a satisfactory representation for the relatively sharp transition which is observed experimentally. A value of $p = -5$ will be arbitrarily chosen yielding

$$Nu_x = 0.032\phi^{4/5} \Bigg/ \left[1 + \frac{7.53 \times 10^{-6}\,\phi_l^5}{\phi^{7/2}}\right]^{1/5} \tag{12}$$

This expression can be simplified somewhat by introducing the upper critical value ϕ_u for transition from turbulent motion, defined as the intersection of Equations (7) and (10) Hence

$$0.032\phi_u^{4/5} = E\phi_u^{3/2} = 0.3387\phi_u^{3/2}/\phi_l \tag{13}$$

Substituting for ϕ_l in Equation (12) from (13), we get

$$Nu_x = 0.032\phi^{4/5} \Bigg/ \left[1 + \left(\frac{\phi_u}{\phi}\right)^{7/2}\right]^{1/5} \tag{14}$$

Equations (5) and (14) may now be combined to give an overall expression for all three regimes. In the interest of simplicity it is convenient to first subtract 0.45 from Nu_x in Equation (14). This value is completely negligible in the range of applicability of Equation (14). The resulting test equation is

$$(Nu_x - 0.45)^s = (0.3387\phi^{1/2})^s + \left[0.032\phi^{4/5} \Bigg/ \left[1 + \left(\frac{\phi_u}{\phi}\right)^{7/2}\right]^{1/5}\right]^s \tag{15}$$

The data for the transition from laminar motion are not precise or consistent enough to define the exponent s unambiguously. However, a value of 2 appears to be generally satisfactory, resulting in

$$\frac{Nu_x - 0.45}{0.3387\phi^{1/2}} = \left[1 + (\phi/2600)^{3/5} \Bigg/ \left[1 + \left(\frac{\phi}{\phi_u}\right)^{7/2}\right]^{2/5}\right]^{1/2} \tag{16}$$

Equations (2), (7), and (16) are compared with several sets of experimental data in Figure 1. Data for the laminar regime itself are somewhat limited, but Equation (2) is clearly a satisfactory lower bound for all of the data. Equation (7) is similarly seen to be a satisfactory upper bound. The data of Sugawara and Sato (1952) for air appear to be in error on the high side for low ϕ and on the low side for higher ϕ, as previously noted by Kestin et al. (1961). The data of Kestin et al. (1961), Reynolds et al. (1958), and Zukauskus and Slančiauskus (1973) for air appear to scatter widely between the laminar and turbulent limits. However, this dispersion is evidently due to the extraneous factors such as free-stream turbulence and surface roughness which determine the point of transition from laminar to turbulent motion. The data of Zukauskus and Slanciauskus for water and various oils follow rather distinct paths of transition and can be represented reasonably well by Equation (16) with the appropriate choice of ϕ_u as indicated. However, these data appear to be somewhat high at low ϕ. Apparently any consistent set of data for a single fluid can be represented by Equation (16) with the appropriate choice of the single coefficient ϕ_u.

MEAN NUSSELT NUMBER

The mean Nusselt number can be obtained by integrating h as given by Equation (17) over x from 0 to x. However, the resulting expression would be very unwieldy. A much simpler expression of adequate accuracy can be obtained simply by replacing the three coefficients in Equation (16) with those corresponding to the integrated mean expressions for the three regimes. Thus 0.3387 is replaced by $2(0.3387) = 0.6674$, 2 600 by

$$[2(0.3387)/5(0.032)/4]^{10/3} = 12\,472,$$

and ϕ_u by ϕ_{um}, the intersection of the equations for the mean Nusselt number in the transition and turbulent regime, yielding

$$\frac{\overline{Nu} - 0.45}{0.6774\,\phi^{1/2}} = \left[1 + (\phi/12\,500)^{3/5} \Big/ \left[1 + \left(\frac{\phi_{um}}{\phi}\right)^{7/2}\right]^{2/5}\right]^{1/2} \qquad (17)$$

It is apparent that $\phi_{um} = (5/4)\ (3/2)\ \phi_u = 1.875\phi_u = 0.06445\phi_l^{10/7} = 0.02148\phi_{lm}^{10/7}$. However, the introduction of ϕ_u, ϕ_l, or ϕ_{lm} in Equation (17) leads to a more complicated expression. The computed values of Dennis and Smith (1966) and several sets of experimental data are compared with Equation (17) in Figure 2. Again, the component parts of Equation (17) for laminar and turbulent flow provide good lower and upper bounds for the data. The paths of transition are not as clearly defined as for Nu_x. Curves for several arbitrary values of ϕ_{um} are included.

CONCLUSIONS

Equations (16) and (17) represent the computed and experimental data for heat transfer by forced convection over isothermal flat plates and along isothermal cylinders within their uncertainty. The use of these equations in the transition regime requires the choice of a coefficient such as ϕ_l, ϕ_{ml}, ϕ_u, or ϕ_{um} which characterizes the point of transition from laminar or turbulent flow, since this point is dependent on other variables such as surface roughness, upstream turbulence, and the history of the flow.

Equations (2) to (16) are for uniform wall temperature. They may be adapted for uniform heat flux by substituting 0.4637 and 0.02052 for 0.3387 and 0.0468. Equation (17) is presumed to be applicable to uniform heat flux as well as uniform temperature if the temperature difference at $x/2$ is used in $\overline{Nu}$.

Equations (16) and (17) are presumed to be applicable to component transfer with Sh and Sc substituted for Nu and Pr.

The effect of physical property variation with temperature is difficult to generalize. Equations (16) and (17)

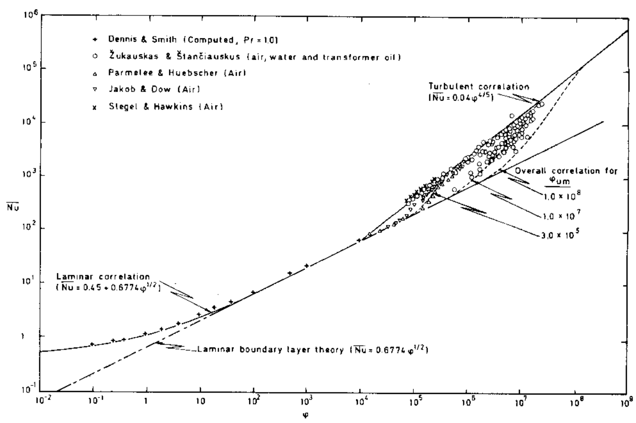

Fig. 2. Correlating equations for mean Nusselt number in forced convection from flat plates.

are presumed to be applicable for a reasonable temperature difference if the properties are all evaluated at the average of the wall and bulk temperature.

The coefficients of 2 600 and 12 500 have some uncertainty related to the uncertainty of the coefficients of 0.032 and 0.04 for completely turbulent flow. The value of $Nu_o = 0.45$ was chosen to provide an empirical representation for the limited calculations of Dennis and Smith and is highly arbitrary. The exponents in Equations (16) and (17) arise from combinations of the exponents of ϕ of 3/2 and 4/5 for transitional and turbulent flow and the arbitrary combining exponents of 5 and 2. The correlation is not particularly sensitive to the choice of the latter two values.

Figures 1 and 2 were plotted only to illustrate the relationship between Equations (16) and (17) and the experimental and computed data. Owing to the wide range of the coordinates, they are not proposed for direct use.

NOTATION

B	=	coefficient for turbulent regime
C	=	central value of Pr, Equation (5)
$\mathcal{D}$	=	diffusivity, m^2/s
E	=	coefficient for transitional regime
j	=	heat flux density, $W/m^2 \cdot s$
$\bar{j}$	=	mean heat flux density from 0 to x, $W/m^2 \cdot s$
k	=	thermal conductivity, $W/m^2 \cdot s \cdot k$
k'	=	mass transfer coefficient, m/s
m	=	exponent for turbulent regime
n	=	combining exponent for laminar regime
Nu_x	=	$jx/k\Delta T$ = local Nusselt number at x
$\overline{Nu}$	=	$\bar{j}x/k\Delta T$ = mean Nusselt number
Nu_o	=	limiting Nusselt number as $x \to 0$
p	=	combining exponent for transition and turbulent regimes
Pr	=	ν/α = Prandtl number
Re	=	$u_\infty x/\nu$ = Reynolds number
s	=	combining exponent for laminar and transition-turbulent regime
Sc	=	$\nu/\mathcal{D}$ = Schmidt number
Sh	=	$k'x/\mathcal{D}$ = Sherwood number
u_∞	=	free stream velocity, m/s
x	=	distance from starting edge, m
α	=	thermal diffusivity, m^2/s
ΔT	=	temperature difference between plate and fluid
ν	=	kinematic viscosity, m^2/s
ϕ	=	$RePr^{2/3}/[1 + (0.0468/Pr)^{2/3}]^{1/2}$

Subscripts

l	=	lower critical value based on Nu_x [at intersection of Equations (2) and (8)]

lm = lower critical value based on $\overline{Nu}$
s = surface
u = upper critical value based on Nu_x [at intersection of Equations (6) and (10)]
um = upper critical value based on $\overline{Nu}$
∞ = bulk fluid

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Heat Transfer and Curing in Polymer Reaction Molding

A theoretical model is proposed for curing in polymer reaction molding operations like casting, potting, lamination, thermoset molding, or reaction injection molding. In many cases, convection and mass diffusion can be neglected. The resulting unsteady heat equation with temperature dependent generation is solved numerically. Temperature and property profiles, gel point, sol fraction, and modulus are presented for a typical urethane system under several conditions. Two dimensionless groups, the adiabatic temperature rise and the ratio of polymerization rate to heat conduction rate, are most significant. Process design criteria are discussed.

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SCOPE

A large number of polymer products are formed into their final shape by polymerization in situ. Examples include processes like monomer casting, potting or encapsulation, bonding with structural adhesives, in situ foaming, reinforced plastic lamination, and thermoset and rubber molding operations. The recent development of a number of very fast urethane systems with a wide choice of final properties has resulted in a new processing method, reaction injection molding (Wood, 1974; DelGatto, 1973), which has high growth potential.

There is a body of descriptive literature on these processes. Problems of nonuniform reaction due to heat transfer and the reaction exotherm are well recognized (for example: acrylics, Horn, 1960; epoxies, Lee and Neville, 1967; polyesters, Doyle, 1969; rubber, Hills, 1971). Some qualitative guidelines for processing are given, but there are few analytical studies. As Kamal (1974) states in a recent review of thermoset injection molding, "So far there has been no systematic study in the literature to relate basic kinetic, thermal and rheological parameters for thermosets to their injection molding behavior." In rubber molding, Hills (1971) reports some transient heat transfer calculations to predict cure development in thick sections. Engel-

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